Math Camp for Economics PhD Students

Rice University

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Course Logistics

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Schedule: May 25 - July 19, MTWThF
Time: 40 hours of prerecorded lectures, accessible anytime
Exam: July 20 - August 2
Location: Canvas Platform, https://canvas.rice.edu
Office Hours: TBD, Canvas

Course Outline

The aim of this course is to introduce or remind you of the mathematics required for Ph.D. courses in economics. In this course, we will learn the standard tools and cookbook procedures that are required for the first year Ph.D. courses.

We divide the lectures into five modules: Real Analysis, Linear Algebra, Calculus, Optimization, and Difference and Differential Equations. There will be one homework for each module, and there will be multiple choice questions to be answered following the online lectures to measure your attendance. You should expect to study two hours on average for reviewing the material and doing the homeworks. The homeworks can be completed individually or in groups of two students. There will also be a quiz on each module, which must be
completed individually and is closed book. Quizzes will typically not be proctored, although we may proctor some quizzes.

There will be an exam at the end of the course, which will be proctored using a webcam. We will schedule time slots with individual students prior to the exam week.

In addition, there will be office hours four times a week. The attendance to the office hours is not mandatory but highly encouraged. We will be available to answer your questions and you will have opportunity to interact with your classmates.

The weights of attendance at the lectures (multiple choice questions), homeworks, quizzes, and the exam are 10%, 40%, 20%, and 30% respectively.

**General Readings**


**Module I. Real Analysis (Lectures 1-4)**

Set theory: sets, algebra of sets, families of sets, cartesian product, binary relations and ordered sets, functions and correspondences, equivalence relation, supremum and infimum.

Metric spaces 1: cardinality, metric spaces and subspaces, open and closed sets, interior, closure and boundary, compact sets, dense and connected sets.

Metric spaces 2: sequences and subsequences, convergence and divergence, Cauchy sequences and their properties, upper and lower limits, complete metric spaces, series and absolute convergence.

Metric spaces 3: continuous functions and alternative characterizations, properties of continuous functions, upper and lower semicontinuity of functions, homeomorphisms, upper and
lower hemicontinuity of correspondences.

Readings:


**Module II. Linear Algebra (Lectures 5-8)**

Vectors and matrices: Cartesian coordinate system, vectors and vector algebra, vector spaces, linear dependence, basis and subspace, matrices and matrix algebra, properties of matrix operations, block matrices.

Systems of linear equations: determinant, inverse, column and row spaces and rank, systems or linear equations, substitution, Gaussian elimination, existence and cardinality of solutions, Cramer’s rule, fundamental theorem of linear algebra.

Convexity: convex sets, convex hull, extreme points, cone, linear and affine functions, convex and concave functions and their properties, quasiconvex and quasiconcave functions and their properties.

Separating hyperplane theorems: perpendicularity, line, hyperplane, separating hyperplane theorems, applications in exchange economies and game theory.

Readings:


Module III. Calculus: (Lectures 9-11)

Differentiation 1: real valued functions, affine functions and polynomials, derivative, properties of differentiable functions, chain rule, mean value theorem, monotonicity and differentiation, inverse function theorem,

Differentiation 2: l’Hopital’s rule, higher order derivatives, convexity and differentiable functions, Taylor’s theorem, derivative of vector valued functions, functions of several variables, total derivative, chain rule, partial derivative, directional derivative.

Differentiation and integration: implicit functions, implicit function theorem, higher order derivatives, convexity of differentiable functions, integration, Riemann integral, integrability, differentiation and integration, integration by parts, Leibniz integral rule.

Readings:


Module IV. Optimization (Lectures 12-17)

Optimization: preferences, utility representation, optimization problem, existence of a maximizer, saddle point theorem.

Linear programming: linear programming problem, activity analysis problem, diet problem, duality theorem, saddle point theorem for linear programming, equilibrium theorem.

Unconstrained optimization: optimization problem, examples of optimization problems, unconstrained optimization, local and global maxima, necessity, sufficiency, convexity and optimization, cookbook procedure,

Inequality constraints: optimization problem, necessity, alternative formulation, global optimum and sufficiency, cookbook procedure.

Parametric continuity and fixed point theorems: Berge’s maximum theorem, envelope theorem, intermediate value theorem, baby-Brouwer fixed point theorem, Brouwer fixed point theorem, Kakutani fixed point theorem, applications.

Readings:


**Module V. Difference and Differential Equations (Lectures 18-20)**

Difference equations: basic concepts, first order difference equations, cobweb diagram, equilibrium and stability, second order difference equation, equilibrium and stability, non-homogeneous second order linear difference equations, system of linear difference equations, equilibrium and stability.

Differential equations: basic concepts, existence and uniqueness of solutions, first order differential equations, graphical solutions, directional field, phase diagram, analytical solutions, second order homogeneous differential equations, second order non-homogeneous differential equations, stability.

Systems of differential equations: basic concepts, graphical solution, solution to linear systems, stability, linear approximation of nonlinear systems.

Readings: